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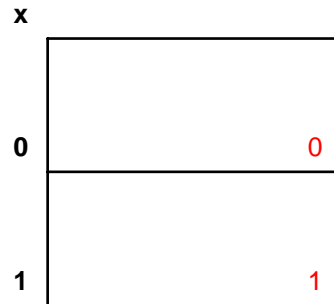
# SIMPLIFICACIÓN DE FUNCIONES CANÓNICAS

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# Mapas de Karnaugh

- Proceso sistemático para la simplificación de expresiones de conmutación.
- Se trata de una matriz de casillas o celdas, cada una de las cuales representa un minitérmino de una FC
- Si FC tiene n-variables  $\Rightarrow 2^n$  casillas.

1 variable



# Mapas de Karnaugh (minitérminos)

## 2 variables

|                      |       |       |
|----------------------|-------|-------|
| $x_0 \backslash x_1$ | 0     | 1     |
| 0                    | 00(0) | 01(1) |
| 1                    | 10(2) | 11(3) |

## 4 variables

|                            |          |          |          |          |
|----------------------------|----------|----------|----------|----------|
| $x_3x_2 \backslash x_1x_0$ | 00       | 01       | 11       | 10       |
| 00                         | 0000(0)  | 0001(1)  | 0011(3)  | 0010(2)  |
| 01                         | 0100(4)  | 0101(5)  | 0111(7)  | 0110(6)  |
| 11                         | 1100(12) | 1101(13) | 1111(15) | 1110(14) |
| 10                         | 1000(8)  | 1001(9)  | 1011(11) | 1010(10) |

## 3 variables

|                         |        |        |        |        |
|-------------------------|--------|--------|--------|--------|
| $x_2 \backslash x_1x_0$ | 00     | 01     | 11     | 10     |
| 0                       | 000(0) | 001(1) | 011(3) | 010(2) |
| 1                       | 100(4) | 101(5) | 111(7) | 110(6) |

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## Mapas de Karnaugh (minitérminos)

- Procedimiento de simplificación. Paso 1
  - ❑ minitérmino  $m_i \Rightarrow$  casilla  $i$
  - ❑ Cubrir todos los minitérminos con el mínimo número de rectángulos posibles.
  - ❑ Construir rectángulos tan grandes como sea posible.
  - ❑ Para simplificar, una casilla se puede cubrir varias veces.
  - ❑ Hay que empezar con las casillas que se pueden cubrir de menos maneras.

# Mapas de Karnaugh (minitérminos)

Ej. 1:  $f(x_3x_2x_1x_0) = \sum m(0, 2, 5, 6, 7, 8, 10, 15)$

$$f(x_3x_2x_1x_0) = \overline{x_3}\overline{x_2}\overline{x_1}\overline{x_0} + \overline{x_3}\overline{x_2}x_1\overline{x_0} + \overline{x_3}x_2\overline{x_1}\overline{x_0} + \overline{x_3}x_2x_1\overline{x_0} + \overline{x_3}\overline{x_2}\overline{x_1}x_0 + \overline{x_3}\overline{x_2}x_1x_0 + \overline{x_3}x_2\overline{x_1}x_0 + \overline{x_3}x_2x_1x_0$$

- Las casillas 5 y 15 sólo se pueden cubrir de una manera  $\Rightarrow$  son las primeras que cubrimos.
- Las esquinas adyacentes van juntas.

| $x_3x_2 \backslash x_1x_0$ | 00           | 01           | 11            | 10            |
|----------------------------|--------------|--------------|---------------|---------------|
| 00                         | 1<br>0000(0) |              |               | 1<br>0010(2)  |
| 01                         |              | 1<br>0101(5) | 1<br>0111(7)  | 1<br>0110(6)  |
| 11                         |              |              | 1<br>1111(15) |               |
| 10                         | 1<br>1000(8) |              |               | 1<br>1010(10) |

- La casilla 6 puede ir con la 2 y con la 7. Elegimos la 7.

# Mapas de Karnaugh (minitérminos)

## ■ Procedimiento de simplificación. Paso 2

- Si tenemos un diagrama para n-variables y creamos rectángulos de  $2^r$  casillas  $\Rightarrow$  (n-r) dígitos iguales = número de variables en la expresión simplificada.

| $x_3x_2$ \ $x_1x_0$ | 00 | 01 | 11 | 10 |
|---------------------|----|----|----|----|
| 00                  | 1  |    |    | 1  |
| 01                  |    | 1  | 1  | 1  |
| 11                  |    |    | 1  |    |
| 10                  |    |    |    |    |

(The table above is a simplified representation of the content in the image. The original image shows a 4x4 Karnaugh map with binary values and decimal indices in the cells. The values are: (0,0)=1, (0,1)=0, (0,2)=0, (0,3)=1, (1,0)=0, (1,1)=1, (1,2)=1, (1,3)=1, (2,0)=0, (2,1)=0, (2,2)=1, (2,3)=0, (3,0)=0, (3,1)=0, (3,2)=0, (3,3)=0. The decimal indices are: (0,0)=0, (0,1)=1, (0,2)=2, (0,3)=3, (1,0)=4, (1,1)=5, (1,2)=6, (1,3)=7, (2,0)=8, (2,1)=9, (2,2)=10, (2,3)=11, (3,0)=12, (3,1)=13, (3,2)=14, (3,3)=15. The map is circled with green, red, and yellow lines to show groupings.)

- $n=4 \wedge r=2 \Rightarrow$  2 variables  $\Rightarrow \overline{x_0}x_2$
- $n=4 \wedge r=1 \Rightarrow$  3 variables  $\Rightarrow$   
5 y 7:  $\overline{x_3}x_2x_0$   
7 y 15:  $x_1x_2x_0$
- $n=4 \wedge r=1 \Rightarrow$  3 variables  $\Rightarrow$   
6 y 7:  $\overline{x_3}x_2x_1$

$$f(x_3x_2x_1x_0) = \overline{x_2}x_0 + \overline{x_3}x_2x_1 + \overline{x_3}x_2x_0 + x_2x_1x_0$$

# Mapas de Karnaugh (minitérminos)

Ej.2:  $f(x_3x_2x_1x_0) = \sum m(0,2,3,5,8,9,12,13,14,15)$

| $x_3x_2 \backslash x_1x_0$ | 00            | 01            | 11            | 10            |
|----------------------------|---------------|---------------|---------------|---------------|
| 00                         | 1<br>0000(0)  | 0<br>0001(1)  | 1<br>0011(3)  | 1<br>0010(2)  |
| 01                         | 0<br>0100(4)  | 1<br>0101(5)  | 0<br>0111(7)  | 0<br>0110(6)  |
| 11                         | 1<br>1100(12) | 1<br>1101(13) | 1<br>1111(15) | 1<br>1110(14) |
| 10                         | 1<br>1000(8)  | 1<br>1001(9)  | 0<br>1011(11) | 0<br>1010(10) |

- $n=4 \wedge r=1 \Rightarrow 3$  variables  $\Rightarrow \overline{x_3} \overline{x_2} x_1$      $\overline{x_3} \overline{x_2} x_0$      $\overline{x_1} x_0 x_2$
- $n=4 \wedge r=2 \Rightarrow 2$  variables  $\Rightarrow x_3 x_2$
- $n=4 \wedge r=2 \Rightarrow 2$  variables  $\Rightarrow \overline{x_3} x_1$

$$f(x_3x_2x_1x_0) = \overline{x_3} \overline{x_2} x_1 + \overline{x_3} \overline{x_2} x_0 + \overline{x_1} x_0 x_2 + x_3 x_2 + \overline{x_3} x_1$$

# Mapas de Karnaugh (minitérminos)

Ej. 3:  $f(x_3x_2x_1x_0) = \sum m(0,1,2,3,5,7,8,9,10,11)$

| $x_3x_2 \backslash x_1x_0$ | 00           | 01           | 11            | 10            |
|----------------------------|--------------|--------------|---------------|---------------|
| 00                         | 1<br>0000(0) | 1<br>0001(1) | 1<br>0011(3)  | 1<br>0010(2)  |
| 01                         |              | 1<br>0101(5) | 1<br>0111(7)  |               |
| 11                         |              |              |               |               |
| 10                         | 1<br>1000(8) | 1<br>1001(9) | 1<br>1011(11) | 1<br>1010(10) |

- $n=4 \wedge r=3 \Rightarrow 1$  variable  $\Rightarrow \overline{x_2}$
- $n=4 \wedge r=2 \Rightarrow 2$  variables  $\Rightarrow \overline{x_3x_0}$

$$f(x_3x_2x_1x_0) = \overline{x_2} + \overline{x_3x_0}$$



# Mapas de Karnaugh (minitérminos)

Ej.4:  $f(x_2x_1x_0) = \sum m(1,3,4,5)$

|                         |         |         |         |         |
|-------------------------|---------|---------|---------|---------|
| $x_2 \backslash x_1x_0$ | 00      | 01      | 11      | 10      |
| 0                       | 0000(0) | 0001(1) | 0011(3) | 0010(2) |
| 1                       | 0100(4) | 0101(5) | 0111(7) | 0110(6) |

Groupings: A red box groups minterms 1 and 3 ( $\bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1x_0$ ). A green box groups minterms 4 and 5 ( $x_2\bar{x}_1\bar{x}_0 + x_2\bar{x}_1x_0$ ). A blue box groups minterms 1 and 5 ( $\bar{x}_2x_1\bar{x}_0 + x_2x_1\bar{x}_0$ ).

- $n=3 \wedge r=1 \Rightarrow 2$  variables  $\Rightarrow \bar{x}_2\bar{x}_0$
- $n=3 \wedge r=1 \Rightarrow 2$  variables  $\Rightarrow \bar{x}_1x_0$
- $n=3 \wedge r=1 \Rightarrow 2$  variables  $\Rightarrow x_1x_2$

$$f(x_2x_1x_0) = \bar{x}_2\bar{x}_0 + \bar{x}_1x_0 + x_1x_2$$

|                         |         |         |         |         |
|-------------------------|---------|---------|---------|---------|
| $x_2 \backslash x_1x_0$ | 00      | 01      | 11      | 10      |
| 0                       | 0000(0) | 0001(1) | 0011(3) | 0010(2) |
| 1                       | 0100(4) | 0101(5) | 0111(7) | 0110(6) |

Groupings: A red box groups minterms 1 and 3 ( $\bar{x}_2\bar{x}_1x_0 + \bar{x}_2x_1x_0$ ). A green box groups minterms 4 and 5 ( $x_2\bar{x}_1\bar{x}_0 + x_2\bar{x}_1x_0$ ).

- $n=3 \wedge r=1 \Rightarrow 2$  variables  $\Rightarrow \bar{x}_2\bar{x}_0$
- $n=3 \wedge r=1 \Rightarrow 2$  variables  $\Rightarrow x_1x_2$

$$f(x_2x_1x_0) = \bar{x}_2\bar{x}_0 + x_1x_2$$

# Simplificación FC incompletamente definidas

Ej.2:  $f(x_3x_2x_1x_0) = \sum m(2,5,8,12,14) + \sum d(0,9,11,13,15)$

| $x_3x_2 \backslash x_1x_0$ | 00      | 01 | 11      | 10      |
|----------------------------|---------|----|---------|---------|
| 00                         | d       |    |         | 1       |
| 01                         | 0100(4) | 1  | 0111(7) | 0110(6) |
| 11                         | 1       | d  | d       | 1       |
| 10                         | 1       | d  | d       |         |

(Note: The table above is a simplified representation of the content in the image. The original image shows the full 4x4 grid with minterm numbers and don't care values 'd'. The grid is as follows:

| $x_3x_2 \backslash x_1x_0$ | 00       | 01       | 11       | 10       |
|----------------------------|----------|----------|----------|----------|
| 00                         | 0000(0)  | 0001(1)  | 0011(3)  | 0010(2)  |
| 01                         | 0100(4)  | 0101(5)  | 0111(7)  | 0110(6)  |
| 11                         | 1100(12) | 1101(13) | 1111(15) | 1110(14) |
| 10                         | 1000(8)  | 1001(9)  | 1011(11) | 1010(10) |

- $n=4 \wedge r=2 \Rightarrow 2$  variables  $\Rightarrow x_3x_2$
- $n=4 \wedge r=2 \Rightarrow 2$  variables  $\Rightarrow x_3x_1$
- $n=4 \wedge r=1 \Rightarrow 3$  variables  $\Rightarrow x_3x_2x_0$        $x_1x_0x_2$

$$f(x_3x_2x_1x_0) = x_3x_2x_0 + x_2x_1x_0 + x_3x_2 + x_3x_1$$

# Mapas de Karnaugh (maxitérminos)

## 2 variables

|       |       |       |       |
|-------|-------|-------|-------|
|       | $x_1$ | 0     | 1     |
| $x_0$ | 0     | 00(3) | 01(2) |
|       | 1     | 10(1) | 11(0) |

## 4 variables

|          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|
|          | $x_1x_0$ | 00       | 01       | 11       | 10       |
| $x_3x_2$ | 00       | 0000(15) | 0001(14) | 0011(12) | 0010(13) |
|          | 01       | 0100(11) | 0101(10) | 0111(8)  | 0110(9)  |
|          | 11       | 1100(3)  | 1101(2)  | 1111(0)  | 1110(1)  |
|          | 10       | 1000(7)  | 1001(6)  | 1011(4)  | 1010(5)  |

# Mapas de Karnaugh (maxitérminos)

Ej.3:  $f(x_3x_2x_1x_0) = \prod(1,3,4,9,11,12)$

| $x_3x_2 \backslash x_1x_0$ | 00       | 01       | 11       | 10       |
|----------------------------|----------|----------|----------|----------|
| 00                         | 0000(15) | 0001(14) | 0011(12) | 0010(13) |
| 01                         | 0100(11) | 0101(10) | 0111(8)  | 0110(9)  |
| 11                         | 1100(3)  | 1101(2)  | 1111(0)  | 1110(1)  |
| 10                         | 1000(7)  | 1001(6)  | 1011(4)  | 1010(5)  |

Diagram showing the Karnaugh map for the function  $f(x_3x_2x_1x_0) = \prod(1,3,4,9,11,12)$ . The map is a 4x4 grid with rows labeled  $x_3x_2$  (00, 01, 11, 10) and columns labeled  $x_1x_0$  (00, 01, 11, 10). The cells contain the binary representation of the minterm and its index in parentheses. The function is 0 for minterms 12, 11, 9, 8, 1, and 4. These 0s are highlighted with blue boxes. Red boxes highlight the 0s at (00,11) and (10,11).

- $n=4 \wedge r=2 \Rightarrow 2$  variables  $\Rightarrow \overline{x_2} + x_0$
- $n=4 \wedge r=1 \Rightarrow 3$  variables  $\Rightarrow x_2 + \overline{x_1} + \overline{x_0}$

$$f(x_3, x_2, x_1, x_0) = (\overline{x_2} + x_0) \cdot (x_2 + \overline{x_1} + \overline{x_0})$$

# Ejercicio 4.5

$$f(x_2x_1x_0) = \sum m(0,6,7) + \sum d(1,2,5) = \overline{x_2}x_1\overline{x_0} + x_2\overline{x_1}\overline{x_0} + x_2x_1x_0$$

| $x_2$ | $x_1$ | $x_0$ | <b>f</b> |
|-------|-------|-------|----------|
| 0     | 0     | 0     | 1        |
| 0     | 0     | 1     | d        |
| 0     | 1     | 0     | d        |
| 0     | 1     | 1     | 0        |
| 1     | 0     | 0     | 0        |
| 1     | 0     | 1     | d        |
| 1     | 1     | 0     | 1        |
| 1     | 1     | 1     | 1        |

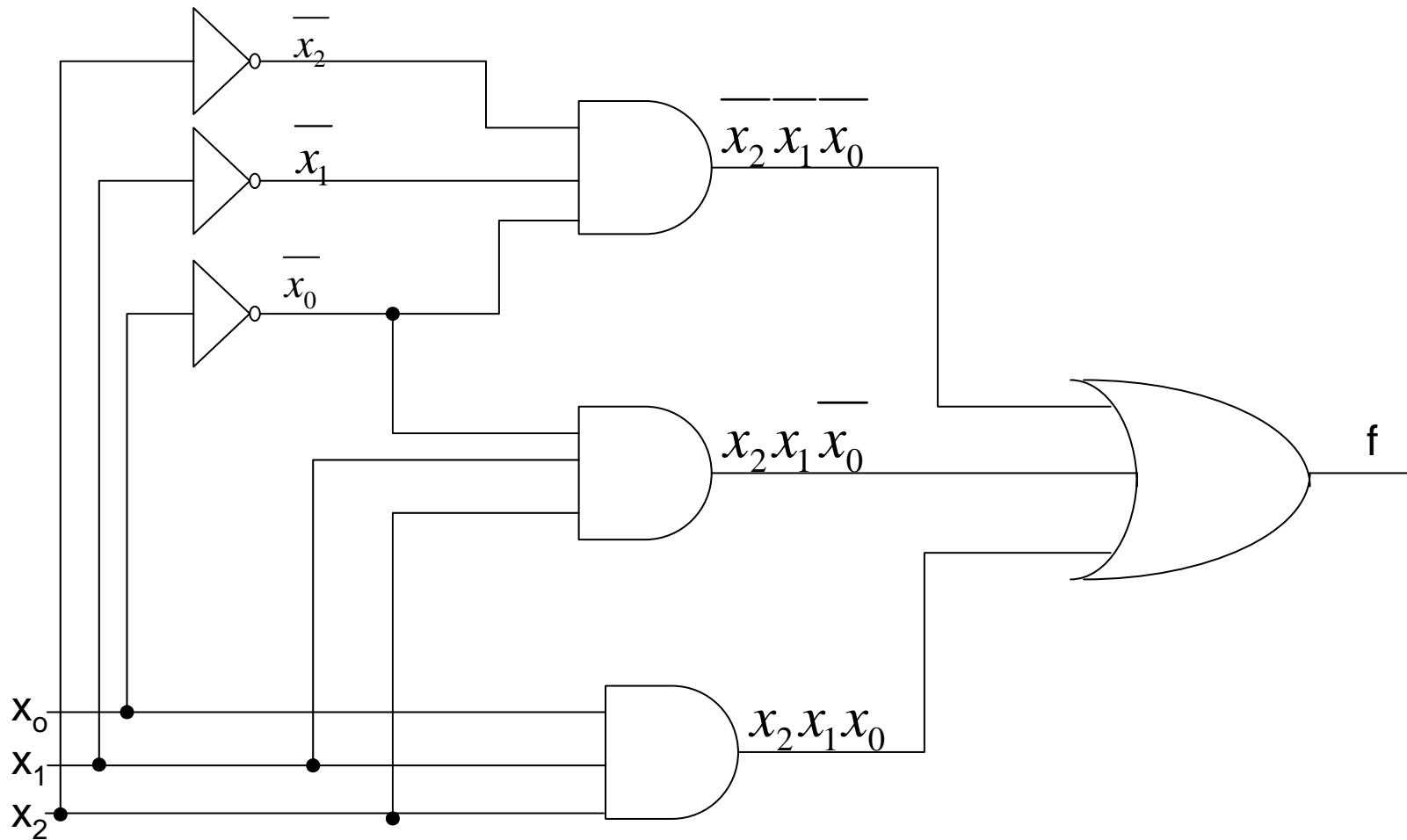
| $x_2 \backslash x_1x_0$ | 00           | 01           | 11           | 10           |
|-------------------------|--------------|--------------|--------------|--------------|
| 0                       | 1<br>0000(0) | d<br>0001(1) | <br>0011(3)  | d<br>0010(2) |
| 1                       | <br>0100(4)  | d<br>0101(5) | 1<br>0111(7) | 1<br>0110(6) |

- $n=3 \wedge r=1 \Rightarrow 2$  variables  $\Rightarrow \overline{x_2}x_0$
- $n=3 \wedge r=1 \Rightarrow 2$  variables  $\Rightarrow x_2x_1$

$$f(x_2x_1x_0) = \overline{x_2}x_0 + x_2x_1$$

## Ejercicio 4.5. Diseño con puertas lógicas

$$f(x_2x_1x_0) = \sum m(0,6,7) + \sum d(1,2,5) = \overline{x_2}\overline{x_1}\overline{x_0} + x_2x_1\overline{x_0} + x_2x_1x_0$$



## Ej.4.5. Diseño simplificado con puertas lógicas

$$f(x_2x_1x_0) = \overline{\overline{x_2x_0}} + x_2x_1$$

